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Tests of Magnetometer/Sun-Sensor Orbit Determination Using Flight Data

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A magnetometer-based orbit determination batch filter has been improved and tested with real flight data from Dynamics Explorer 2 (DE-2), the Magnetic Field Satellite (MAGSAT), and the Ørsted satellite. These tests have been conducted to determine the performance of a low-cost autonomous orbit determination system. The spacecraft's orbit, magnetometer biases, and correction terms to the Earth's magnetic field are estimated by this filter. Synthesized sun-sensor data, in addition to real magnetometer data, are used to make the field model correction terms observable. The filter improvements include a new dynamics model and consideration of a priori variances for the field model corrections. The best performance that has been achieved with this filter is a maximum position error of 4.48 km for a 2-day batch of DE-2 data with second-order/degree field model corrections and 2.35 km for a 24-h batch of MAGSAT data with second-order/degree field model corrections. The maximum position error for a 24-h batch of Ørsted data is 59.50 km without field model corrections, but is only 2.19 km with 10th-order/degree field model corrections.

I. Introduction

ORBIT determination is an old topic in celestial mechanics and is an essential part of satellite navigation. Traditional ground-based tracking methods that use range and range-rate measurements can provide an orbit accuracy to within a few centimeters.[1] Autonomous orbit determination using only onboard measurements can be a requirement of military satellites to guarantee independence from ground facilities.[2] The rapid increase in the number of satellites also increases the need for autonomous navigation because of bottlenecks in ground tracking facilities.[3]

A filter that uses magnetometer measurements provides one possible means of performing autonomous orbit determination. This idea was first introduced by Psiaki and Martel[4] and has been tested by a number of researchers since then.[3,5–9] Magnetometers fly on most spacecraft (S/C) for attitude determination and control purposes. Therefore, successful autonomous orbit determination using magnetometer measurements can make the integration of attitude and orbit determination possible and lead to reduced mission costs. Although magnetometer-based autonomous orbit determination is unlikely to have better accuracy than ground-based tracking, a magnetometer-based system could be applied to a mission that does not need the accuracy of ground-station tracking. The Tropical Rainfall Measurement Mission (TRMM), for example, requires 40-km position accuracy. An integrated magnetometer-based attitude and orbit determination system can also be used as a backup system. Some of examples of backup operation are profile recovery of a mission when the spacecraft loses its orientation, antenna pointing in the case of loss of contact with the ground station, and checks of the primary orbit and attitude system to warn of abnormal behavior.

Ground-based tests of such systems that used real flight data reported position accuracies in the range from 8 to 125 km (Refs. 3 and 6–9). Psiaki et al. conjectured that some of the inaccuracy was caused by uncertainty in the Earth's magnetic field.[6] To combat this source of uncertainty, batch filters have been developed that estimate field model corrections in addition to the orbit of the S/C.[10,11] Either three-axis star sensor data[10] or sun-sensor data,[11] in addition to the magnetometer data, were added in these studies to make the orbit and the field model corrections simultaneously observable. These studies used truth-model simulation to test their designs, and they predicted improvements in the accuracy of magnetometer-based orbit determination. Their predicted position accuracies ranged down to better than 1 km.

The main goal of this work is to test the efficiency of the field model correcting magnetometer-based orbit determination filter by using real flight data. This is the first such test. The particular filter that has been tested is based on the one that uses magnetometer and sun-sensor data, as in Ref. 11. This work also improves the orbit propagation dynamics model over what has been used in Refs. 4–6 and 10 and 11, and it considers a priori variances for the field model corrections.

The rest of this paper consists of three main sections plus conclusions. Section II explains how the batch filter of Ref. 11 has been improved for use in this study. Section III describes the spacecraft data that have been used in this study. The test results with real flight data are presented in Sec. IV.

II. Modifications of Batch Filter

The field model correcting orbit determination batch filter of Ref. 11 is extensively used in this study. To deal with real flight data, the orbital dynamics model had to be improved. The cost function has also been changed to include a priori variances for the field model corrections, which allow the filter to decide whether it has enough data to estimate accurately these corrections. This section explains the orbital dynamics model improvements and gives a review of the batch filter that has been used in this study. The addition of a priori variances for the field model corrections is explained in the review of the batch filter.

Note that the batch filter is not practical for autonomous orbit determination. The main focus of this study, however, is to prove the validity of the field model correction concept. Therefore, a batch filter that was already developed in Ref. 11 to use magnetometer and sun-sensor data is used extensively. Development of a Kalman filter version could be done in the future once the benefit of the field model correcting approach has been verified.

A. Update of Orbital Dynamics Model

The orbital dynamics model that was used in the Ref. 11 filter considered only secular J_2 effects and the effects of drag on altitude and mean motion. It ignored the periodic effects of J_2, higher-order gravity effects, and the drag effects on eccentricity. A more accurate dynamics model is preferable for dealing with real flight data.

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The new dynamics model used in this study considers the full J_2 effects, and it uses a drag model that is based on the 1976 U.S. standard atmosphere.[12] The orbit is calculated via direct numerical integration of the following equation of motion, which is expressed in an Earth-centered Earth-fixed (ECEF) reference frame:

$$\ddot{\boldsymbol{r}} + 2\boldsymbol{\omega} \times \dot{\boldsymbol{r}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) = \boldsymbol{a}_{\text{inertial}} = -\left(\mu_\oplus \big/ \|\boldsymbol{r}\|^3\right)\boldsymbol{r} + \boldsymbol{a}_{J2}(r) + \boldsymbol{a}_d(r, \dot{\boldsymbol{r}}) \tag{1}$$

where $\boldsymbol{\omega}$ is the Earth's rotation rate vector, $\mu_\oplus$ is the geocentric gravitational constant, $\boldsymbol{r}$ is the position vector of the S/C in the ECEF frame, $\boldsymbol{a}_{J2}(\boldsymbol{r})$ is the gravitational acceleration due to Earth oblateness, and $\boldsymbol{a}_d(\boldsymbol{r}, \dot{\boldsymbol{r}})$ is the acceleration due to drag. Note that the ECEF reference frame is defined with $+x$ pointing through the equator at the Greenwich meridian and $+z$ pointing through the north pole. The standard fourth-order Runge–Kutta method with a fixed integration step size is used in the integration of Eq. (1), where $\boldsymbol{a}_{J2}(\boldsymbol{r})$ is[13]

$$\boldsymbol{a}_{J2}(\boldsymbol{r}) = \frac{9}{2}\frac{\mu_\oplus}{\|\boldsymbol{r}\|^5} J_2 R_\oplus^2 \left[\left(\frac{z}{\|\boldsymbol{r}\|}\right)^2 - \left(\frac{1}{3}\right)\right]\boldsymbol{r} + 3\frac{\mu_\oplus}{\|\boldsymbol{r}\|^7} J_2 R_\oplus^2 z \begin{bmatrix} zx \\ zy \\ -(x^2 + y^2) \end{bmatrix} \tag{2}$$

where $R_\oplus$ is the Earth's equatorial radius; J_2 is the gravity field's lowest zonal harmonic coefficient; and x, y, and z are the Cartesian coordinates of the S/C in the ECEF frame, $\boldsymbol{r} = [x, y, z]^T$. Here, $\boldsymbol{a}_d(\boldsymbol{r}, \dot{\boldsymbol{r}})$ is

$$\boldsymbol{a}_d(\boldsymbol{r}, \dot{\boldsymbol{r}}) = -(\bar{\beta}/2)\rho(\boldsymbol{r})\|\dot{\boldsymbol{r}}\|\dot{\boldsymbol{r}} \tag{3}$$

where ρ is the atmospheric mass density, $\bar{\beta} = C_D S/m$ is the inverse of the S/C's ballistic coefficient, C_D is the S/C's drag coefficient, S is its aerodynamic reference area, and m is its mass. Here, $\rho(\boldsymbol{r})$ is calculated from a cubic spline interpolation of the natural log of tabulated data from the 1976 U.S. standard atmosphere.[12]

The accuracy of the filter's new dynamics model has been checked and compared to that of the old dynamics model. This has been done by using another batch filter to estimate the six initial conditions for this dynamics model plus $\bar{\beta}$ by minimizing the sum of the square errors between the measured position time history of an actual S/C and the modeled position based on propagation of Eq. (1). The closeness of this fit gives an indication of the model's propagation accuracy.

This dynamics evaluation filter has been run on real flight data from Dynamics Explorer 2 (DE-2) and the Magnetic Field Satellite (MAGSAT). The maximum position error of this test when using the old dynamics model is 2.16 km for a 24-h MAGSAT data set and 4.35–4.90 km for 48-h DE-2 data sets. The new dynamics model has a smaller maximum position error than the old one: 1.6 km for a 24-h MAGSAT data set and 3.50–4.33 km for 48-h DE-2 data sets. Thus, the new dynamics model has a slightly better orbital propagation accuracy than the old one and is accurate enough for use in the magnetometer-based navigation filter.

Note that the constraints in computing resources for autonomous operation are not severe considering the recent dramatic improvements in computer power. Therefore, the improved dynamics model is probably too simple to be used in a typical resource-rich computing environment. The goal of this study, however, is to verify the contribution of the field model corrections to the accuracy improvement of magnetometer-based orbit determination using real flight data. Therefore, a very accurate dynamics model that includes higher-order perturbations of the Earth's gravity field and perturbations from the sun or the moon has not been used in this study.

B. Estimation Vector

The batch filter's estimation vector with the new dynamics model is

$$\boldsymbol{p} = \left[x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0, \bar{\beta}, b_x, b_y, b_z, q_1^0, q_1^1, s_1^1, \dot{q}_1^0, \dot{q}_1^1, \dot{s}_1^1, \dot{g}_1^0, \dot{g}_1^1, \dot{h}_1^1, \Delta g_1^0, \Delta g_1^1, \Delta h_1^1, \Delta g_2^0, \Delta g_2^1, \Delta h_2^1, \Delta g_2^2, \Delta h_2^2, \Delta g_3^0, \ldots, \Delta h_N^N\right]^T \tag{4}$$

where the first 6 elements are the initial position and velocity of the S/C in the ECEF frame, the 7th element is the inverse of the ballistic coefficient, the 8th–10th elements constitute a magnetometer bias vector, and the remaining elements are correction terms to coefficients of a spherical harmonic expansion of the Earth's magnetic field.[10,11] The field correction elements include an external ring current field model (the first six correction terms) and perturbations to the Earth's internal magnetic field model. Note that N is the maximum order and degree of the field model corrections.

C. Batch Filter

The batch filter used in this study is the same as the one used in Ref. 11, except for an improvement to the dynamics model and an addition to the cost function that models a priori variances of the field model corrections. It finds the $\boldsymbol{p}$ estimation vector that best approximates the measurement data based on the dynamics and measurement models.

Two pseudomeasurements, the magnitude of the Earth's magnetic field and the cosine of the angle between the Earth's magnetic field vector and the sun direction vector, are used. These measurements are independent of the S/C's attitude. Let $\boldsymbol{B}_{\text{mes}(k)}$ be the magnetic field vector that is measured by the magnetometer and let $\hat{\boldsymbol{s}}_{\text{mes}(k)}$ be the sun unit direction vector that is measured by the sun sensor, with both measurements occurring at sample time t_k. Both of these measurements are expressed in a common S/C-fixed coordinate system. Then the two pseudomeasurements are given by

$$y_{1\text{mes}(k)}(\boldsymbol{p}) = \sqrt{(\boldsymbol{B}_{\text{mes}(k)} - \boldsymbol{b})^T(\boldsymbol{B}_{\text{mes}(k)} - \boldsymbol{b})} \tag{5a}$$

$$y_{2\text{mes}(k)}(\boldsymbol{p}) = \frac{\hat{\boldsymbol{s}}_{\text{mes}(k)}^T(\boldsymbol{B}_{\text{mes}(k)} - \boldsymbol{b})}{y_{1\text{mes}(k)}(\boldsymbol{p})} \tag{5b}$$

where $\boldsymbol{b} = [b_x, b_y, b_z]^T$ is the estimated magnetometer bias vector expressed in S/C coordinates.

The modeled values of the two pseudomeasurements are

$$y_{1\,\text{mod}}(t_k; \boldsymbol{p}) = \sqrt{\boldsymbol{B}_{\text{sez}}^T \boldsymbol{B}_{\text{sez}}} \tag{6a}$$

$$y_{2\,\text{mod}}(t_k; \boldsymbol{p}) = \frac{\hat{\boldsymbol{s}}_{\text{ECIF}}^T(t_k) \cdot (A_{\text{ECIF/sez}} \cdot \boldsymbol{B}_{\text{sez}})}{y_{1\,\text{mod}}(t_k; \boldsymbol{p})} \tag{6b}$$

where $\boldsymbol{B}_{\text{sez}}[\boldsymbol{r}(\boldsymbol{p}, t_k), q_1^0, q_1^1, \ldots, \Delta h_N^N, t_k]$ is the modeled Earth's magnetic field vector in local south–east–zenith coordinates, $\hat{\boldsymbol{s}}_{\text{ECIF}}(t_k)$ is the sun unit vector in an Earth-centered inertially fixed (ECIF) reference frame, and $A_{\text{ECIF/sez}}[\boldsymbol{r}(\boldsymbol{p}, t_k), t_k]$ is the transformation matrix from local south–east–zenith coordinates to ECIF coordinates. $\boldsymbol{B}_{\text{sez}}$ is calculated using a spherical harmonic expansion and includes the effects of the estimated field model corrections.[10]

The nonlinear least-squares cost function that gets minimized by the batch filter is

$$J(\boldsymbol{p}) = \frac{1}{2}\sum_{k=1}^{M}\left[\frac{y_{1\,\text{mod}}(t_k; \boldsymbol{p}) - y_{1\text{mes}(k)}(\boldsymbol{p})}{\sigma_B}\right]^2 + \frac{1}{2}\sum_{k=1}^{M}\left[\frac{y_{2\,\text{mod}}(t_k; \boldsymbol{p}) - y_{2\text{mes}(k)}(\boldsymbol{p})}{\sigma_{y2(k)}}\right]^2 + \frac{1}{2}\left[\left(\frac{\Delta g_1^0}{\sigma_{g_1^0}}\right)^2 + \left(\frac{\Delta g_1^1}{\sigma_{g_1^1}}\right)^2 + \left(\frac{\Delta h_1^1}{\sigma_{h_1^1}}\right)^2 + \left(\frac{\Delta g_2^0}{\sigma_{g_2^0}}\right)^2 + \left(\frac{\Delta g_2^1}{\sigma_{g_2^1}}\right)^2 + \left(\frac{\Delta h_2^1}{\sigma_{h_2^1}}\right)^2 + \cdots + \left(\frac{\Delta h_N^N}{\sigma_{h_N^N}}\right)^2\right] \tag{7}$$

where M is the number of data samples; σ_B is the standard deviation of the field magnitude measurement; $\sigma_{y2(k)}$ is the standard deviation of the cosine pseudomeasurement, which is described in Ref. 11; and the σ_g and σ_h quantities are a priori standard deviations of the spherical harmonic coefficients of the Earth's internal magnetic

Table 1 Sample coefficients of the 1995 IGRF field model and the corresponding nominal a priori variances (σ_g or σ_h) for the field model corrections

Model/variance, nT	g_1^0	g_1^1	h_1^1	g_2^0	g_2^1	h_2^1
1995 IGRF model	−29682	−1789	5318	−2197	3074	−2356
σ_g or σ_h	91.00	57.00	119.50	73.00	10.00	10.25

field model. The first and second terms are the ones that are used in Ref. 11, and the remaining terms are added in this study to model a priori knowledge of the field model coefficients. These additions help to ensure reasonable field model corrections. The second term of the cost function is calculated only for sample times when sun-sensor data are available.

The nominal σ_g and σ_h values of Eq. (7) have been chosen as the average of two observed five-year changes in the corresponding coefficients of the International Geomagnetic Reference Field (IGRF) model. One change was for the period 1980–1985, and the other change was for 1995–2000. Note, however, that a value of 0.5 nT has been used whenever the average change was less than this lower bound. A special 1980 Earth magnetic field model or the 1995 or 2000 IGRF model has been used as the a priori field model in each of this study's various tests. When the measurement date of real flight data is close to the starting date of an a priori magnetic field model, $\frac{1}{10}$ of the nominal σ_g and σ_h values have been used to allow only smaller variances in the field model corrections and to achieve a more reasonable predicted accuracy from a consider-covariance analysis. Nominal σ_g and σ_h values are used for measurement data that are measured far from the starting date of the a priori magnetic field model. Such cases need larger field model corrections. Table 1 shows the first few terms of the spherical harmonic coefficients of the 1995 IGRF model and the corresponding nominal σ_g or σ_h values. The coefficients of Table 1 are used with associated Legendre functions of degree n and order m that are normalized according to the convention of Schmidt (see Ref. 13).

The Gauss–Newton iterative numerical method is used to solve this nonlinear least-squares problem.[11] It requires the calculation of the partial derivatives of the measurement errors with respect to the $\boldsymbol{p}$ vector, which it uses to find search directions. They are also used to compute the filter's predicted position accuracy using consider-covariance analysis.

Consider-covariance takes into account the effects of unestimated parameters on the filter accuracy.[14] The systematic errors due to magnetic field model coefficients that do not get estimated are included in a consider-covariance analysis. Systematic errors of unmodeled perturbations to the dynamics model, such as unmodeled higher-order gravity terms and solar radiation effects, are not considered in the consider-covariance analysis in this study.

D. Alternate Filter That Does Not Estimate Field Model Corrections

An alternate batch filter has been developed to provide a point of comparison for the field model correcting filter. It does not estimate field model corrections, nor does it use sun-sensor measurements. It only considers the first term of the cost function in Eq. (7). This filter is similar to the batch filter of Ref. 6, except that Ref. 6 used the poorer dynamics model mentioned earlier. This filter can be used to evaluate the advantages of field model corrections over the traditional magnetometer-based orbit determination without field model corrections.

E. Additional Filters

Two additional batch filters have been developed for test purposes. One is a field model correcting filter with an a priori drag coefficient estimate. It is assigned a relatively small a priori drag variance. This filter keeps its estimated $\bar{\beta}$ nearly fixed at the a priori value of 0.03 m^2/kg by adding an additional term to the cost function of Eq. (7) that minimizes the weighted square error between $\bar{\beta}$ and its a priori value. This filter is useful for a higher-altitude S/C (perigee altitude above the 500–600 km range) because $\bar{\beta}$ becomes nearly unobservable due to the minimal effects of drag at such altitudes.

The other filter is a field model correcting filter that does not estimate orbit. This filter uses tracked position instead of the estimated S/C orbit and only estimates field model correction terms. This filter can help to determine whether the accuracy of field model correction estimates is compromised by the simultaneous estimation of orbit.

III. Real Flight Data

Real flight data from three S/C, DE-2, MAGSAT, and Ørsted, have been used to test the batch filter. These S/C are described hereafter. Data from a fourth S/C, the TRMM, also have been tested. Its magnetometer data, however, were uncalibrated, and the per-axis residual error standard deviations were over 300 nT. These poorly calibrated data led to large maximum position errors for the orbit determination filter, on the order of 50 km. Therefore, the results for TRMM are not included in this study because an accurately calibrated magnetometer is necessary if one wants to apply this technique.

A. DE-2

The DE-2 spacecraft was launched in August 1981 to study the coupling between the magnetosphere, the ionosphere, and the upper atmosphere. DE-2 had roughly a 290-km perigee altitude, a 810-km apogee altitude, and a high inclination, 89.9 deg. It operated for about two years. Magnetometer data, three-axis attitude data derived from sensors, and position data derived from traditional range and range-rate ground-based tracking are all available for DE-2. DE-2 is known to have a 28-nT 1-σ accuracy for the magnetometer data and better than 1-km accuracy for the position data. Sun-sensor data were not directly available. Therefore, it has been synthesized by transforming the known sun position in ECIF coordinates using the DE-2 estimate of the attitude transformation matrix from ECIF coordinates to S/C coordinates. The attitude uncertainty of this transformation is about 0.3 deg. The position of the sun in the ECIF coordinate system is calculated using the algorithm in Ref. 15, which provides a 0.006-deg sun position accuracy. Eclipse by the Earth is also considered in synthesizing sun-sensor data.

Tests have been run using two sets of data extracted from a CD-ROM archive of the DE-2 data that are available from the National Space Science Data Center (NSSDC). One set was measured on 1–2 November 1981 (data set A), and the other set was measured on 15–17 March 1982 (data set B). Both data sets were measured when the activity index for solar magnetic storms was low. The sample period is 60 s, and both data sets are spread out over about 48 h. There are, however, data gaps about every 2 or 3 h, and some of these gaps are as long as 10 h. Thus, the actual measurement coverage time is 13.9 and 6.7 h for data sets A and B, respectively. Bad magnetometer data points have been removed before testing the batch filter. A bad point is considered to occur when the measured magnetometer data is off by over 1000 nT from the modeled magnetometer data based on the known S/C position from ground tracking. Two out of 835 data points in set A and 30 out of 433 data points in set B have been edited out.

The position tracking data of DE-2 is given in altitude, latitude, and longitude and is stated to be measured in geocentric coordinates assuming a spherical Earth. In the process of checking the accuracy of the filter's new dynamics model, however, it has been found that the position data must have been measured with respect to an ellipsoidal Earth, not with respect to a spherical Earth. Therefore, it is assumed that the tracking data of DE-2 have been measured with respect to the ellipsoidal World Geodetic Survey 1984 (WGS-84) model, which is used in describing the Earth as an ellipsoid throughout this study.

B. MAGSAT

The MAGSAT spacecraft was launched in October 1979. The objective of this mission was to obtain accurate vector measurements of the near-Earth geomagnetic field for field modeling purposes. The orbital properties were 97-deg inclination, 330-km perigee height, and 500-km apogee height. This S/C had a cesium scalar magnetometer for calibration purposes and a vector fluxgate magnetometer. The accuracy of the vector magnetometer was determined to be 3 nT, 1-σ after inflight calibration using the scalar magnetometer. The three-axis magnetic field components and the tracked position of the S/C using range and range-rate data are available in the CD-ROM archive of the NSSDC. The accuracy of tracked position is 60 m radially and 300 m horizontally (along track). The magnetic field vector data are expressed in south, east, zenith local coordinates. Because of the imperfect calibration of the attitude determination system, frequent small jumps occur in the vector magnetometer data, but not in the magnitude of the measured magnetic field. Two MAGSAT data sets have been used. One set was measured on 2 February 1980 (data set A), and the other set was measured on 12 March 1980 (data set B). The sampling period of all three data sets was about 100 s without any gaps, and it was measured on magnetically quiet days. Each data set lasts 24 h.

Sun-sensor data have been synthesized for data sets A and B. This is similar to what has been done for DE-2, except that the $y_{2\mathrm{mes}}$ dot product is computed in south, east, zenith local level coordinates instead of in S/C coordinates.

C. Ørsted

The Danish satellite Ørsted was launched in February 1999. The main objective of this mission is to provide a precise global mapping of the Earth's magnetic field. The orbit of Ørsted had an apogee altitude of 865 km, a perigee altitude of 649 km, and an inclination of 96.48 deg as of 10 February 2000. Ørsted has a scalar magnetometer and a fluxgate vector magnetometer, and both magnetometers are placed on an 8-m deployable boom. Absolute position data and vector magnetometer data in local south, east, zenith coordinates are available. The position of the S/C is determined by an onboard global positioning system (GPS) receiver and has an accuracy on the order of 2–8 m. The vector magnetometer data are calibrated using the scalar magnetometer for scale factor, bias, and nonorthogonality of the axes and then are calibrated for the transformation to S/C coordinates, which are the axes of the star imager. This alignment transformation is accurate to 0.003 deg after calibration. The accuracy of the vector magnetometer has been determined to be less than 3 nT 1-σ after in-flight calibration. The accuracy of the star imager is 0.003 deg perpendicular to its boresight and 0.014 deg about its boresight. The vector magnetometer data are transformed from S/C coordinates to local south, east, zenith coordinates using attitude data. Attitude information, however, is not explicitly available in the data archive of Ørsted. This is the same as for the MAGSAT data sets.

Four data sets that were measured on 21–22 December 1999 (data sets A and B) and 18–19 January 2000 (data sets C and D) have been tested. All data sets were measured when the solar storm activity level was low. The data were sampled at about 30-s intervals and have a maximum data gap of 57 min. Each data set lasts 24 h, and the last 3 h of data sets A and C overlap with the first 3 h of data sets B and D, respectively. These overlapped data sets are used for orbit overlap tests that evaluate the precision of the filter's estimated position. Similar to MAGSAT, sun-sensor measurements and the $y_{2\mathrm{mes}}$ dot product have been synthesized in local south, east, zenith coordinates.

IV. Results of Filter Tests Using Flight Data

Two batch filters, one without field model corrections and the other with field model corrections, have been tested with DE-2, MAGSAT, and Ørsted flight data. The filter without field model corrections uses only magnetometer data, and the one with field model corrections uses both magnetometer and sun-sensor data. The magnetometer bias vector is estimated in both filters only for DE-2 because this is the only S/C whose magnetic field vector data are given in S/C coordinates. It would not make physical sense to define a bias in the local south, east, zenith coordinate system in which the MAGSAT or Ørsted data are available. Tests of the filter without field model corrections can be compared with tests of the field model correcting filter to check whether there is an advantage to including field model corrections.

To use the batch filter that estimates field model corrections along with the orbit, sun-sensor data are used in addition to magnetometer data. When using mixed data types in a filter, it is important to use reasonable predictions of their accuracies to weight properly their errors, as in Eq. (7). The 1-σ accuracy of the magnetometer data and the sun-sensor data are assumed to be 15 nT and 0.15 deg, respectively, for all three S/C. With these σ values as inputs, the filter's optimal residual measurement errors show similar levels of accuracy. MAGSAT and Ørsted have attitude accuracies that are about 10 times better than this sun-sensor σ, but this inconsistency is allowable because Ref. 11 indicates that the filter's performance is relatively insensitive to this σ value.

The performance of these filters has been evaluated by comparing their S/C position estimates with those that have been derived from ground-tracking data or an onboard GPS sensor. The filters' predicted accuracies of their position estimates also have been calculated using the estimation vector's consider-covariance and linearized covariance propagation techniques.[10,14] This predicted accuracy has been compared with the actual accuracy as a means of checking the validity of the consider analysis.

A. DE-2 Results

Langel and Estes's 80 model (see Ref. 16) is used as the a priori field model for the DE-2 data. Langel and Estes's 80 model was derived using the data from MAGSAT. The secular variation coefficients of this model are considered to be valid between 1978 and 1982, and the magnetic field in DE-2's 1981–1982 time frame is calculated by using the secular variations of this model. The Langel and Estes's 80 model only depends on measurements that were taken up to 1980. Therefore, Langel and Estes's 80 model would have been available during the operation period of DE-2 and, thus, is a reasonable model to be used in the test of an operational orbit determination filter. Because the measurement time of DE-2 data sets are not far from the starting date (1980) of the a priori field model, $\frac{1}{10}$ of the nominal σ_g and σ_h values of Table 1 are used in considering a priori variances of field model coefficients.

The DE-2 results for these batch filters are summarized in Table 2. Table 2 presents the results for data sets A and B. Note that Table 2 has 11 rows and 7 columns. The first column gives the value of N, the highest order and degree of the field model corrections that the filter estimates [review Eq. (4)]. Thus, each row except the first row corresponds to a different maximum order and degree of the filter's estimated field model corrections, and the complexity of the

Table 2 Position error metrics for two different DE-2 data sets

	Data set A			Data set B		
Maximal order and degree of field model corrections	Predicted maximum 1-σ accuracy of position, km	Maximum position error magnitude, km	rms value of position error, km	Predicted maximum 1-σ accuracy of position, km	Maximum position error magnitude, km	rms value of position error, km
No corrections	——	5.29	3.94	——	5.87	3.68
First	3.54	5.33	2.27	5.11	8.16	4.77
Second	3.06	4.48	2.54	5.36	9.08	5.72
Third	2.05	5.76	2.82	3.32	6.99	4.21
Fourth	1.36	6.58	3.11	3.07	8.19	4.23
Fifth	1.32	7.93	3.76	2.92	8.47	4.79
Sixth	1.30	8.17	3.85	2.87	7.61	4.20
Seventh	1.27	7.97	3.47	2.87	7.93	4.11
Eighth	1.25	8.08	3.54	2.87	7.77	4.13
Ninth	1.25	8.13	3.54	2.87	7.74	4.11
Tenth	1.25	8.17	3.54	2.87	7.76	4.13

Table 3 Position error metrics for two different MAGSAT data sets

Maximal order and degree of field model corrections	Data set A: Predicted maximum 1-σ accuracy of position, km	Data set A: Maximum position error magnitude, km	Data set B: Predicted maximum 1-σ accuracy of position, km	Data set B: Maximum position error magnitude, km
No corrections	——	7.02	——	4.56
First	4.07	3.88	4.62	3.24
Second	3.40	4.04	3.54	2.35
Third	3.07	3.54	3.22	2.78
Fourth	1.45	3.15	1.56	3.29
Fifth	1.32	3.00	1.34	3.42
Sixth	1.23	2.86	1.26	3.38
Seventh	1.19	3.07	1.22	3.41
Eighth	1.18	2.97	1.18	3.28
Ninth	1.18	3.00	1.18	3.26
Tenth	1.18	3.00	1.17	3.23

corrections (and of the filter) increases as one moves from the upper rows to the lower rows. (Note that in all cases considered in this paper the filter's field model uses a 10th-order/10th-degree field model. It uses a priori coefficients for terms that do not get corrected by the filter's estimation process.) The first row corresponds to the filter that does not estimate field model corrections and that uses only magnetometer data. The second and fifth columns tabulate the consider-analysis predictions of the maximum standard deviations of the position error magnitude for the two different cases. Unless otherwise noted, the term maximum refers to maximization of a given quantity over the entire batch interval. Columns 3 and 6 give the actual maximum position error magnitudes for these two cases as computed by differencing the estimated positions with the true positions as derived from ground-based tracking or GPS data. Columns 4 and 7 present the rms of the position error magnitudes.

The maximum position error magnitude without field model corrections is 5.29 km for data set A and 5.87 km for data set B. These results represent an improvement of a factor of three or more from the DE-2 results of Ref. 6. This improvement is thought to be the result of an improved calibration of the DE-2 data.

The position error magnitudes associated with data set A are smaller than the corresponding values for data set B for lower-order/degree corrections. With first–fourth order/degree field model corrections, data set A yields maximum position error magnitudes from 4.48 to 6.58 km, but data set B's maximum position error magnitude ranges from 6.99 to 9.08 km. For higher-order/degree field model corrections, maximum position error looks similar for both data sets, but the rms position errors show that data set A has better position accuracy than data set B.

The better position accuracy of data set A may come from the actual magnetometer data coverage time of data set A being two times longer than that of data set B: 13.9 h for data set A vs 6.7 h for data set B. This effect is probably the reason why data set A yields lower predicted 1-σ position accuracies (compare columns 2 and 5 of Table 2).

Overall, the results with field model corrections are slightly worse than those without field model corrections. Even the best-case data set A results with field model corrections (maximum position error of 4.48 km) are only slightly better than the data set A results without these corrections (maximum position error of 5.29 km). The best-case B results with corrections (maximum position error of 6.99 km) are not as good as the results for the filter that does not estimate field model corrections (maximum position error of 5.87 km). These results may be caused by the low percentage of actual measurement coverage time for DE-2 over the two-day span of each data set.

Note that the dynamics model used in this study can cause a maximum position error of about 4 km over the 48-h duration of a DE-2 data set: 3.50 km for data set A and 4.33 km for data set B. Therefore, a maximum position error for magnetometer-based orbit determination as low as 4.48 km is very good. It is almost at the limit of the accuracy of the dynamics model.

The filter's predicted maximum position standard deviation improves as the order/degree of the corrections increases for lower-order/degree corrections and keeps similar values for sixth- or higher-order/degree corrections. The actual maximum position error, however, does not get better with higher-order/degree corrections. Therefore, rough estimation of the actual position error of the batch filter based on its consider-analysis position accuracy may not be good for DE-2 cases.

B. MAGSAT Results

All MAGSAT data used in this test were measured in 1980. Therefore, Langel and Estes' 80 model (see Ref. 16) has been used for the MAGSAT data sets.

Two batch filters, one without field model corrections and the other one with field model corrections, have been tried for data sets A and B. Table 3 summarizes the MAGSAT results for these filters. These results assume $\frac{1}{10}$ of the nominal σ_g and σ_h values as 1-σ accuracies of the field model coefficients.

The best maximum position error magnitudes for the field model correcting filter are 2.86 km for data set A with sixth-order/degree field model corrections and 2.35 km for data set B with second-order/degree field model corrections. These results represent a factor of two or better improvement in the position accuracy for best-case field model corrections in comparison to the accuracy without field model corrections for both data sets. Thus, unlike DE-2, field model corrections can improve the performance of magnetometer-based orbit determination for MAGSAT. It is also true that the results for all order/degree corrections for MAGSAT are better than the results when corrections are not used. The ratios of the maximum position error magnitudes to the consider-analysis predicted maximum position standard deviations range from 0.95 to 2.58 for data set A and from 0.66 to 2.80 for data set B. This shows that the statistical model of the consider-analysis is reasonable. Therefore, the maximum position error of the batch filter can be inferred from the predicted position accuracy of the consider analysis.

C. Ørsted Results

There is a question of whether the DE-2 and MAGSAT results were helped by the data coming from a time period for which there is a very good a priori estimate of the Earth's magnetic field. This is true because the currently available field models for their flight time frames are based on data that extend after the fact, or they are based on MAGSAT's accurate survey of the Earth. To clarify this issue, the orbit determination systems should be tested with recent magnetometer data that are measured when the only available Earth magnetic field model is one that is based only on past data of moderate accuracy. Ørsted data are useful for such a test because Ørsted carries an accurate magnetometer and is currently operational.

Unlike the DE-2 and MAGSAT cases, which have a much lower perigee altitude, $\bar{\beta}$ is not practically observable for Ørsted because the effects of drag on its orbit are minimal. Therefore, the filter that incorporates an a priori $\bar{\beta}$ estimate has been used on Ørsted

Table 4 Position error metrics for Ørsted data sets (December 1999 data) with a priori field model from 1995 IGRF

Maximal order and degree of field model corrections	Data set A: Predicted maximum 1-σ accuracy of position, km	Data set A: Maximum position error magnitude, km	Data set B: Predicted maximum 1-σ accuracy of position, km	Data set B: Maximum position error magnitude, km	Three-h overlap of data sets A and B: Maximum position difference magnitude, km	Three-h overlap of data sets A and B: rms value of position difference, km
No corrections	——	59.50	——	50.71	24.89	22.32
First	34.51	58.72	36.55	54.63	17.34	16.38
Second	41.97	29.14	38.80	26.48	10.29	9.31
Third	17.95	17.55	16.38	19.07	7.81	6.69
Fourth	10.74	8.87	11.49	10.82	7.74	6.35
Fifth	4.28	5.64	4.44	7.39	6.63	5.14
Sixth	3.50	2.41	2.85	3.79	4.49	3.16
Seventh	2.35	2.48	1.86	3.21	3.56	2.62
Eighth	1.13	2.61	1.29	3.40	2.81	1.90
Ninth	1.11	2.46	1.16	3.85	3.10	2.18
Tenth	1.07	2.19	1.15	4.14	3.27	2.15

Table 5 Position error metrics for Ørsted data sets (January 2000 data) with a priori field model from 2000 IGRF

Maximal order and degree of field model corrections	Data set C: Predicted maximum 1-σ accuracy of position, km	Data set C: Maximum position error magnitude, km	Data set D: Predicted maximum 1-σ accuracy of position, km	Data set D: Maximum position error magnitude, km	Three-h overlap of data sets C and D: Maximum position difference magnitude, km	Three-h overlap of data sets C and D: rms value of position difference, km
No corrections	——	9.09	——	7.52	12.63	8.58
First	3.38	7.90	3.77	9.00	2.60	1.79
Second	3.13	5.88	3.24	5.98	2.33	1.45
Third	1.61	5.87	1.67	5.52	1.81	1.34
Fourth	1.16	5.69	1.10	5.08	2.09	1.30
Fifth	0.83	5.52	0.79	5.26	1.78	1.10
Sixth	0.82	5.87	0.78	5.40	2.22	1.39
Seventh	0.81	5.55	0.77	5.24	2.15	1.36
Eighth	0.81	5.38	0.77	5.22	2.02	1.27
Ninth	0.81	5.37	0.77	5.21	2.03	1.28
Tenth	0.81	5.36	0.77	5.21	2.00	1.26

data when orbit and, optionally, field model correction terms are estimated.

Tables 4 and 5 present the results for Ørsted data sets for different orders/degrees of the field model corrections. Results for the filter that does not correct the field model are also shown in the first row of both Tables 4 and 5. Table 4 is for measurement data from December 1999 (data sets A and B), and the filter uses the 1995 IGRF model as its a priori field model. Table 5 is for measurement data from January 2000 (data sets C and D), and the filter's a priori field model is the 2000 IGRF model. The a priori field models have been calculated via propagation of the 1995 or 2000 IGRF model using secular terms. These models have been obtained from the National Geophysical Data Center (http://www.ngdc.noaa.gov/seg/potfld/tab1igrf.shtmlIGRF95.html and ftp://www.ngdc.noaa.gov/Solid_Earth/Mainfld_Mag/Models).

Tables 4 and 5 show the predicted position accuracy from the consider-analysis (second and fourth columns) and the maximum position error (third and fifth columns) for two data sets. The sixth column gives the maximum magnitude of the difference between the two data sets' position estimates during their 3-h overlap period. The seventh column gives the rms of the position difference magnitude for the overlap period. The results in Table 4 used nominal σ_g and σ_h values, whereas the ones in Table 5 used one-tenth of the nominal σ_g and σ_h values. The use of different σ_g and σ_h values is motivated by the fact that higher a priori variances of the field model corrections should be allowed for a very inaccurate a priori field model, such as the one used for the results of Table 4.

The maximum position error magnitude for the filter without field model corrections is 59.50 km for data set A, 50.71 km for data set B, 9.09 km for data set C, and 7.52 km for data set D. The large error for data sets A and B is not surprising because of the large residual error in the a priori field model. The rms residual field model error for data sets A and B from a comparison of raw magnetic field data with the 1995 IGRF a priori magnetic field model is 72.22 nT for data set A and 65.14 nT for data set B. These residual errors are far bigger than the corresponding errors for data sets C and D, 22.52 nT for data set C and 20.84 nT for data set D, when compared with the 2000 IGRF a priori magnetic field model.

Table 4 shows the results for the case when an inaccurate a priori field model is applied. The overall trends of the results for data sets A and B in Table 4 are quite similar. The position estimation error of the field model correcting filter keeps improving as the complexity of the order/degree of the field model corrections increases. For data set A, the maximum position error keeps decreasing from 58.72 km at 1st-order/degree field model corrections down to 2.19 km at 10th-order/degree field model corrections. For data set B, the error in the estimated position reaches a minimum 3.21 km for seventh-order/degree field model corrections. The results of Table 4 suggest that field model corrections of at least sixth- or higher-order/degree should be tried when an inaccurate a priori field model is used. Figure 1 shows the position error magnitude time histories for the best position estimate with field model corrections and for the one without field model corrections for data set A. The field model corrections cause a dramatic improvement of the accuracy of the estimated position. Therefore, the inaccuracy of the a priori Earth magnetic field model seems to be the driving factor in causing position estimation errors in the case of Ørsted data if one uses a filter that does not estimate corrections to the Earth's magnetic field.

The results of Table 5 are less dramatic than the ones of Table 4 because a more accurate a priori field model is used for the results of Table 5. The best position error is 5.36 km with 10th-order/degree

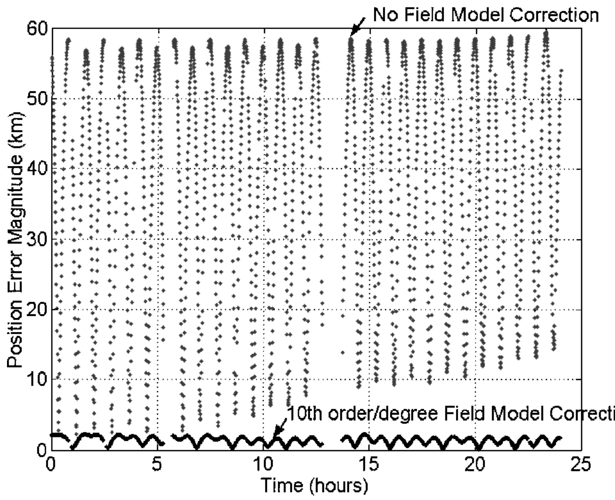

Fig. 1 Position error magnitude time histories for Ørsted data set A with two batch filters.

corrections for data set C and 5.08 km with 4th-order/degree corrections for data set D. The results for third- or higher-order degree corrections remain almost the same for both data sets.

The Ørsted results confirm that the field model correcting batch filter can contribute to a substantial improvement in the orbit determination performance when the only available Earth magnetic field model is inaccurate due to the use of predicted secular variations over a long time scale, that is, 5 years. This conclusion has been further confirmed by a set of erroneous runs that has been conducted in the course of this research. By mistake, Ørsted data set C was originally run using a propagation of the 2000 IGRF field model to 2010. This mistake caused very poor position estimation performance when no field model corrections were estimated or when only low-order corrections were estimated. When high-order corrections were estimated, the position estimation errors become very small, comparable to those that have been obtained using the correct model. Thus, the field model correcting filter was able to correct the IGRF model dating bug.

Tables 4 and 5 also show that the maximum position error is consistent with the filter's predicted position accuracy and with the overlap test results. Compare column 2 with column 3 and column 4 with column 5 to see the consistency of the consider-based accuracy predictions with the actual errors. Compare columns 6 and 7 with columns 3 and 5 to see the consistency of overlap results with the actual errors. In a real autonomous application, maximum position error cannot be calculated by comparing estimated position with ground-station tracked position. The results of Tables 4 and 5 confirm that the combined use of predicted position accuracy from consider-covariance analysis and overlap test results can provide a solid means of estimating position accuracy autonomously and determining a reasonable order/degree to use for the field model corrections.

D. Tests of the Uniqueness and Reality of the Corrected Field Model

The following questions arise: Are the filter's field model corrections unique, and are they indicative of real variations in the Earth's magnetic field? These questions are answered through three tests of the field model correcting filter with Ørsted data set C, which was measured in January 2000. These tests compare various corrected and uncorrected field models. Note that the field model correction terms from the field model correcting filter include corrections to the Earth's internal magnetic field and external ring current effects. Therefore, when a corrected field model is compared to an a priori propagated IGRF field model, only internal terms are considered. If two different corrected field models are compared, then both internal and external terms are considered.

The first test is to check whether the corrected field is unique, which amounts to an observability test. Two different a priori field models, the 1995 IGRF model and the 2000 IGRF model, have been used in the filter, and 10th-order/degree corrections are estimated in each case. The a priori variance terms for the field model corrections are not included in the filter's cost function in this test because the a priori variances incline the filter to generate a corrected field that is close to the a priori field, which is not appropriate for an observability test. The rms value of the magnitude of the field vector error between the two resultant corrected field models has been computed along the Ørsted orbit. This value can be viewed as being a norm-type metric of the discrepancy between the two field models. It is 6.7×10^{-5} nT, which is very small. The high level of agreement between the two different corrected field models implies that the corrected field model is unique and, therefore, simultaneously observable with orbit. Although the small rms error of two resultant corrected fields implies observability, this is only a necessary condition for the corrected field model to capture actual field variations.

The second test checks whether the filter's field model corrections cause the field model to become more accurate. This test compares the magnetic field vector based on the propagated 2000 IGRF field model without corrections with one based on the propagated 1995 IGRF field model. The propagated 2000 IGRF model should be fairly accurate for the January 2000 time frame of data set C; thus, it is used as a sort of truth model for comparison purposes. In one comparison, the propagated 1995 IGRF model is uncorrected, and in the other case it is corrected by the filtering of data set C with nominal values of σ_g and σ_h. The rms value of the magnitude of the field vector error between the propagated 1995 IGRF model and the 2000 IGRF model is 95.87 nT along the orbit before corrections get applied to the 1995 model. This rms error decreases as higher and higher orders and degrees of corrections are estimated and applied to the 1995 model. It decreases to 31.72 nT for 10th-order/degree corrections. This test shows that the field model correcting filter with higher-order/degree corrections drives the inaccurately predicted January 2000 field of the propagated 1995 field model toward the more realistic 2000 IGRF field model. It also confirms that low-order/degree corrections are not enough to predict the January 2000 field model from the 1995 model; for example, first-order/degree corrections left a 93.23 nT rms error.

The third test considers whether the simultaneous estimation of orbit has a negative effect on the estimation of field model corrections. In this test, corrections to the 2000 IGRF field model are estimated for Ørsted data set C in two different ways. One way uses the field model correcting orbit determination filter. The other way uses the second filter that is described in Sec. II.E, the one that estimates only the corrections, not the orbit. Recall that this filter uses the tracking data to determine instantaneous S/C position. The corrected fields that have been estimated by these two techniques have been compared. The rms magnitude of their vector difference is 18.37 nT along the orbit. This small difference indicates that the simultaneous estimation of orbit does not have a large impact on the accuracy of the corrected field model.

The rms errors between the various field models show similar trends in the entire altitude range from 300 to 900 km. The propagated 1995 IGRF model with estimated 10th-order/degree corrections is three to four times closer to the uncorrected 2000 IGRF model than is the propagated, uncorrected 1995 IGRF model. Thus, the field model corrections from the Ørsted orbit determination filter are applicable to a broad range of altitude beyond that of the Ørsted trajectory.

These tests confirm that the field model corrections from the orbit determination filter are unique and that they provide reasonable estimates of the true magnetic field.

E. Summary Results

The performance of the field model correcting filter for different order/degree corrections is different for the three S/C. The maximum position error decreases a lot for data sets A and B of Ørsted as the order/degree of the field model corrections increases. The results for Ørsted data sets C and D and for both MAGSAT data sets improve modestly for lower-order/degree corrections, but then do not vary much as the order/degree of the corrections increases further. They do, however, maintain low position estimation errors for higher-order/degree corrections. DE-2 performance varies from Ørsted and MAGSAT and degrades slightly as the size of the order/degree of the corrections increases. Nevertheless, the position errors are reasonably small for all cases that use sixth-order/degree or higher field model corrections. They are no larger than 8.2 km, which is sufficiently accurate for a number of missions.

The trends of the maximum position error of the field model correcting filter with different order/degree corrections may imply the following. Very high-order/degree field model corrections improve the performance of filter significantly for an inaccurate magnetic field model like the one used for data sets A and B of Ørsted (propagation of the 1995 IGRF model to December 1999). The sixth- or higher-order/degree field model corrections are enough to assure good performance of the filter. If, however, the a priori model is more accurate (like the 2000 IGRF model used in data sets C and D of Ørsted and the 1980 field model used for MAGSAT), then lower-order/degree corrections are enough to achieve good accuracy. In this latter case, higher-order/degree corrections will produce similar performance as with lower-order/degree corrections because the higher-order correction terms will not be very significant. In the DE-2 cases, however, the position estimation becomes a little worse with higher-order/degree corrections, and the field model correcting filter does not necessarily improve position estimation accuracy. This may be due to DE-2's small measurement coverage time of magnetometer data with frequent and long data gaps.

Some facts about the magnetometer data used in this study should be noted. First, all tests are done with magnetometer data on magnetically quiet days. The field model correcting filter may not be effective when solar storm activity is high. This system, however, might still be applied during magnetically active days by the propagation of estimated position from the data of magnetically quiet days. It might help to extend the magnetic field model to include the effect of solar winds, though this addition will make for a more complicated system. Second, the magnetometer data of the three S/C are very accurate, especially for MAGSAT and Ørsted, which have their magnetometers mounted on separate booms. The field model correcting filter has not been tested on a mission with a less accurate magnetometer. In general, the magnetometer data should be clean, with minimum S/C-generated magnetic fields. This might require the magnetometer to be mounted on a separate boom, away from the main S/C body. It may also be necessary to discard magnetometer data that are taken when the local magnetic field changes abnormally due to interference from specific electric equipment. The failure of the filter with uncalibrated TRMM data also suggests that the magnetometer data should be well calibrated. Calibration may include alignments, scale factors, biases, and torquer coupling matrices.

V. Conclusions

Several magnetometer-based orbit determination batch filters have been developed and tested using real flight data. The basic filter estimates a S/C's orbit along with magnetometer biases and, optionally, correction terms to a spherical harmonic model of Earth's magnetic field. If field model corrections are estimated, then the filter requires sun-sensor data as well as magnetometer data. The enhancement to these filters as compared to previous work has been to use an improved orbital dynamics model, to consider a priori variances of field model corrections, and to allow the use of an a priori drag parameter estimate for a higher altitude S/C.

The filter performs well for flight data from the DE-2, MAGSAT, and Ørsted S/C. When only magnetometer data are used, without field model corrections, the maximum position error is 5.29 and 5.87 km for two DE-2 data sets and 7.02 and 4.56 km for two MAGSAT data sets. This filter's maximum position error for one Ørsted data set is 59.50 km. This large error is mainly due to errors in the a priori Earth magnetic field model. Another Ørsted data set yields a 9.09-km maximum position error largely because it was given a better a priori field model.

The performance of the batch filter with Earth's magnetic field model corrections varies slightly among the different S/C. This filter augments the magnetometer data with sun-sensor data to make the field model corrections observable. The addition of field model corrections does not make significant improvements in the case of DE-2, and the addition of high-order corrections slightly degrades the DE-2 position estimation accuracy. The performance with MAGSAT data improves by as much as a factor of two with lower-order/degree corrections. Higher-order corrections also maintain low position errors for MAGSAT. Ørsted data shows the most benefit from the estimation of field model corrections. In one case, the peak position estimation errors are decreased from 59.50 km, when no field model corrections are estimated, down to 2.19 km when corrections up to 10th-degree/order are estimated. In a case with a better a priori Earth magnetic field model, the improvements due to the estimation of field model corrections are less dramatic. The general trends of the MAGSAT and Ørsted results suggest that it would be wise to include field model corrections up to order and degree six if there is any significant uncertainty in the a priori Earth's magnetic field. The DE-2 results imply that the estimation of field model corrections will not work as well if there are long gaps in the magnetometer or sun-sensor data.

The predicted accuracy from the consider-covariance analysis and orbit overlap tests is consistent with the actual maximum position errors. Therefore, these metrics can be used to check the orbit solution accuracy autonomously and to determine a reasonable maximum order/degree of the field model corrections.

The uniqueness and realism of the corrected magnetic field model has been tested using an Ørsted data set measured on January 2000. The corrected field model is unique for 10th-order/degree field model corrections regardless of the a priori field model. Estimation of corrections to a propagated 1995 IGRF field model using data from January 2000 reduces the rms value of the magnitude of field vector differences from the 2000 IGRF field model. The rms difference is 95.87 nT before the field corrections, but it is only 31.72 nT after one applies the filter's estimated field model corrections. This shows that the field model correcting orbit determination filter produces realistic field model perturbations.

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References

[1]Tapley, B. D., Ries, J. C., Davis, G. W., Eanes, R. J., Schutz, B. E., Shum, C. K., Watkins, M. M., Marshall, J. A., Nerem, R. S., Putney, B. H., Klosko, S. M., Luthcke, S. B., Pavlis, D., Williamson, R. G., and Zelensky, N. P., "Precision Orbit Determination for TOPEX/POSEIDON," *Journal of Geophysical Research*, Vol. 99, No. C12, 1994, pp. 24,383–24,404.

[2]Chory, M. A., Hoffman, D. D., and LeMay, J. L., "Satellite Autonomous Navigation—Status and History," *Proceedings of the IEEE Position, Location, and Navigation Symposium*, Inst. of Electrical and Electronics Engineers, New York, 1986, pp. 110–121.

[3]Shorshi, G., and Bar-Itzhack, I.Y., "Satellite Autonomous Navigation Based on Magnetic Field Measurement," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 4, 1995, pp. 843–850.

[4]Psiaki, M. L., and Martel, F., "Autonomous Magnetic Navigation for Earth Orbiting Spacecraft," *Proceedings of the Third Annual AIAA/USU Conference on Small Satellites*, Utah State Univ., Logan, UT, 1989.

[5]Fox, S. M., Pal, P. K., and Psiaki, M. L., "Magnetometer-Based Autonomous Satellite Navigation (MAGNAV)," *Guidance and Control 1990; Proceedings of the Annual Rocky Mountain Guidance and Control Conference*, American Astronautical Society, San Diego, CA, 1990, pp. 369–382.

[6]Psiaki, M. L., Huang, L., and Fox, S. M., "Ground Tests of Magnetometer-Based Autonomous Navigation (MAGNAV) for Low-Earth-Orbiting Spacecraft," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 1, 1993, pp. 206–214.

[7]Wiegand, M., "Autonomous Satellite Navigation via Kalman Filtering of Magnetometer Data," *Acta Astronautica*, Vol. 38, Nos. 4–8, 1996, pp. 395–403.

[8]Deutschmann, J., Harman, R., and Bar-Itzhack, I., "An Innovative Method for Low-Cost, Autonomous Navigation for Low-Earth-Orbit Satellites," *Proceedings of the 1998 AIAA/AAS Astrodynamics Specialist Conference*, AIAA, Reston, VA, 1998, pp. 1–8.

[9]Deutschmann, J., and Bar-Itzhack, I., "Evaluation of Attitude and Orbit Estimation Using Actual Earth Magnetic Field Data," *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 3, 2001, pp. 616–623.

[10]Psiaki, M. L., "Autonomous Orbit and Magnetic Field Determination Using Magnetometer and Star Sensor Data," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 3, 1995, pp. 584–592.

[11]Psiaki, M. L., "Autonomous Low-Earth-Orbit Determination from Magnetometer and Sun Sensor Data," *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 2, 1999, pp. 296–304.

[12]*U.S. Standard Atmosphere*, 1976, National Oceanic and Atmospheric Administration, Washington, DC, 1976.

[13]Wertz, J. R. (ed.), *Spacecraft Attitude Determination and Control*, Reidel, Boston, 1978, pp. 123–129, 779–782.

[14]Biermann, G. J., *Factorization Methods for Discrete Sequential Estimation*, Academic Press, New York, 1977, pp. 162–171.

[15]Russell, C. T., "Geophysical Coordinate Transformations," *Cosmic Electrodynamics*, Vol. 2, No. 2, 1971, pp. 184–196.

[16]Langel, R. A., "The Main Field," *Geomagnetism*, edited by J. A. Jacobs, Vol. 1, Academic Press, New York, 1987, pp. 249–512.